Appendix to "Efficacy of Unilateral Defense in International Crises" in *International Relations*, vol.181 (September, 2015)

Shoko Kohama*

1 The Basic Model: Proof of Proposition 1

1. Pooling on *stand firm*:
   Suppose that textbfT$_h$ and $\mathbf{T}_l$ play *stand firm*, the challenger's posterior belief is $p = \frac{1}{2}$. Then, she fights if

   $$\frac{1}{2}(-\omega_h + \epsilon) + \frac{1}{2}(1 - \omega_l + \epsilon) > 0$$
   $$\Rightarrow \ \epsilon > \frac{1}{2}(\omega_h + \omega_l - 1)$$

   This inequality always holds because $\omega_h + \omega_l < 1$ and $\epsilon \geq 0$. Because the target is not extremely strong in military power, the challenger always chooses to *fight*.
   The target of both types has no incentive to deviate from *stand firm* given the challenger's strategy, because $\omega_h > \omega_l > 0$. Thus, (*stand firm*, *stand firm*), (*fight*), $p = \frac{1}{2}$ is the perfect Bayesian equilibrium.

2. Other cases:
   The target of both types always deviates from *acquiesce* because *acquiesce* is strictly dominated by *stand firm*.

Thus, (*stand firm*, *stand firm*), (*fight*), $p = \frac{1}{2}$ is the unique perfect Bayesian equilibrium of the game. ■

2 The Model of Unilateral Defense: Proof of Proposition 2

1. Pooling on *stand firm*:
   Suppose that the target pools on *stand firm*, the challenger's posterior belief is $p = \frac{1}{2}$. Then, she fights if

   $$\frac{1}{2}(-\omega_h + \alpha + \epsilon) + \frac{1}{2}(1 - \omega_l + \alpha + \epsilon) > 0$$
   $$\Rightarrow \ \epsilon > \frac{1}{2}(\omega_h + \omega_l - 1) - \alpha$$

   This inequality always holds because $\omega_h + \omega_l < 1$, $\alpha > 0$, and $\epsilon \geq 0$. That is, the challenger always *fights*. Then, the low type $\mathbf{T}_l$ deviates from *stand firm* to *acquiesce*, because $\omega_l < \alpha$.

*Hokkaido University, skohama[at]juris.hokudai.ac.jp

2. Pooling on *acquiesce*:
   Suppose that $\mathbf{T}_h$ and $\mathbf{T}_l$ play *acquiesce* in an equilibrium. Off the equilibrium path, the challenger fights if

$$p(-\omega_h + \alpha + \epsilon) + (1-p)(1 - \omega_l + \alpha + \epsilon) > 0$$
$$\Rightarrow \ \epsilon > p(\omega_h + \omega_l - 1) + \omega - \alpha$$

   This inequality always holds because $\omega_h + \omega_l < 1$, $\omega_l < \alpha$, $\epsilon \geq 0$, and $p \geq 0$. That is, the challenger always *fights*. Then, $\mathbf{T}_h$ deviates from *acquiesce* to *stand firm* because $\omega_h > \alpha$.

3. Separation, with $\mathbf{T}_h$ playing *stand firm*:
   Suppose that $\mathbf{T}_h$ plays *stand firm* and $\mathbf{T}_l$ plays *acquiesce*, the challenger's posterior belief is $p = 1$. Then, the challenger's strategy is as follows: she *fights* if $\epsilon > \omega_h - \alpha$, and *back down*, otherwise.
   Because ϵ is uniformly distributed between 0 and 1, the probability π that she fights *ex ante* of the realization of ϵ is $\pi = 1 - \omega_h + \alpha$. Therefore, the expected utility of $\mathbf{T}_h$ is $U_h = \pi(\omega_h - \alpha) + (1-\pi) > 0$ because $\omega_h > \alpha$ and $\pi > 0$. That is, $\mathbf{T}_h$ has no incentive to deviate. Next, the expected utility of $\mathbf{T}_l$ given the challenger's strategy is $U_l = \pi(\omega_l - \alpha) + (1-\pi)$. Recalling that $\omega_l < \alpha$, $\lim_{\alpha \to \omega_h} U_l < 0$. Thus, there is a set of parametric values where U_l is smaller than zero so that $\mathbf{T}_l$ does not mimic the high type.
   For example, when $\omega_h = 0.8$, $\omega_l = 0.1$, and $\alpha = 0.7$, $\pi = 0.9$ and hence, $U_l = -0.44$, which is smaller than the payoff obtained by *acquiesce*.

4. Separation, with $\mathbf{T}_h$ playing *acquiesce*:
   Suppose that $\mathbf{T}_h$ plays *acquiesce* and $\mathbf{T}_l$ plays *stand firm*, the challenger's posterior belief is $p = 0$. Then, the challenger chooses to *fight* because $1 - \omega_l + \alpha + \epsilon > 0$ due to $\omega_l < \alpha$ and $\epsilon \geq 0$. Then, $\mathbf{T}_h$ mimics $\mathbf{T}_l$ to obtain $\omega_h - \alpha > 0$.

Thus, there is a set of parametric values where (*stand firm*, *acquiesce*), (*fight* if $\epsilon > \omega_h - \alpha$ and *back down*, otherwise), $p = 1$ is the unique perfect Bayesian equilibrium of the game. ∎